**Graph Theory Fall 2020**

**Assignment 6**

**Due at 5:00 pm on Friday, October 23**

1. **Show that for any , any tree with a vertex of degree must have at least leaves. The proof that uses summations of the result that a tree always has two leaves is probably easiest to adapt here. You will want to assume in the summations**

To show that the tree must have k leaves, let there by n vertices with one vertex of degree of k and let l be the number of leaves.

By alternation, the rest of the vertices will be having a degree of at least 2 as: .

As stated above, we know that that there will be (n-1) edges in the given tree.

-1)

−l-1) + k

**Therefore,** there will be at least k leaves in any tree with a vertex of degree k.

1. **Suppose is a binary tree of height with vertices.**
2. **As a function of , what are the minimum and maximum possible values of ?**

T is a binary tree, which is a tree data structure in which each of the nodes have at most 2 children, (left and right). It also has the minimum number of nodes given the same fixed height, when every node only has ONE child.

As a function of H, the maximum possible values of n would be the geometric sum .

As a function of H, the minimum possible values of n would be

1. **Suppose every parent has exactly two children in . Show that must be odd.**

Supposing every parent has exactly two children in T, the children of each parent node will be at height H. Let us take the number of nodes at height H as “k”.

As per the given statement, every parent has two i.e. even. Therefore, the number of nodes at such a height (H-1) will be k/2. Computing the sums of all of the nodes at such a height will be .

This can be in a geometric progression which will evaluate to:

The last part of the equation will evaluate to 1 which shows that number of nodes will be odd.

1. **Let be a rooted tree.**
2. **Show that if where is the height of , then and are non-parents.**

IF H is the height of T, then to show D(u,v) = 2H, we need to initially show u and v as parents and proof by contradiction. To show as parents, we will be introducing t1 and t2 as the children of u and v.

The root may or may not be in main path. If we take the path concurred as

Therefore, if then the distance from t1 to the root node will be which is contradicting the height of the tree and as parents. Hence, u and v are non-parents.

1. **Show that is the sum of the levels of and if and only if is on the unique -path.**
2. **Suppose is a simple graph (no loops; no parallel edges) with vertices and edges. What are the possible values for , the number of components of ?**

We are given that; G is a simple graph with n = 14 vertices and m = 7 edges.

As there can be 7 pairs of vertices, there will be 7 **disconnected** edges. Whereas in a simple graph with 7 edges will be containing at least 5.

Therefore, the rest will be G components. In conclusion, the possible values of k are 7 and

1. **Suppose is a binary tree with vertices. What are the minimum and maximum possible values of , the height of ?**

We are given T is a binary tree with .

As we know, the maximum possible values of H are given by, (n-1). Therefore, the maximum height will be

The minimum height of the binary tree is given by the equation . If we substitute the number of vertices (n), we get which will give . Hence the minimum height will be 29.